Convexity of bonds

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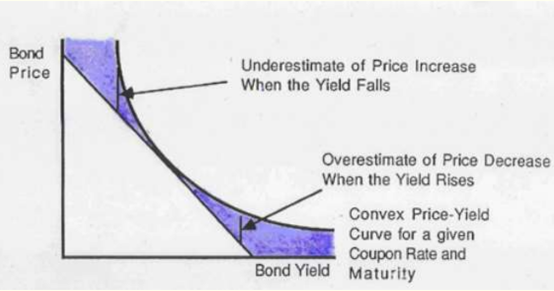
The relationship between the price of bonds and the rate of return has the course of a convex line. For this reason, when we use the duration and the modified duration estimating the price of a bond with a significant change in the rate of return it is very inaccurate (because it assumes a linear course of the price and the rate of return). The measure that allows to eliminate this inaccuracy is **convexity** (it approximates the deviation from the linear return-price curve).

The relationship between the price of bonds and the rate of return



Calculation of convexity if interest is paid once a year

$$C = 0{,}5 \times \frac{\sum_{t=1}^{n} \frac{t \times (t+1) \times C_t}{(1+YTM)^t}}{P \times (1+YTM)^2}$$

C – convexity
P – bond price
C_t – cash flow from bond
YTM – yield to maturity
t – the period in which the payment of interest takes place

Calculation of convexity if interest is paid more than once a year

$$C = 0{,}5 \times \frac{\sum_{t=1}^{n} \frac{t \times (t+1) \times C_t}{(1+\frac{YTM}{m})^t}}{P \times (1+\frac{YTM}{m})^2} \div m^2$$

C – convexity
P – bond price
C_t – cash flow from bond
YTM – yield to maturity
t – the period in which the payment of interest takes place
m – number of interest payments during the year

The formula for the estimation of the convexity of zero-coupon bonds

$$C = \frac{0{,}5 \times n \times (n+1)}{(1+YTM)^2}$$

C – convexity
n – total number of periods of payments
YTM – yield to maturity

The formula for calculating the approximate percentage change of the price of bonds when the rate of return changes on time to maturity

$$\frac{P_1 - P_0}{P_0} = -MD \times (YTM_1 - YTM_0) + C \times (YTM_1 - YTM_0)^2$$

P_0 – the price of bond before the change of rate of return
P_1 – the price of bond after the change of rate of return
MD – modified duration
YTM_0 – yield to maturity before the change
YTM_1 – yield to maturity after the change
C – convexity